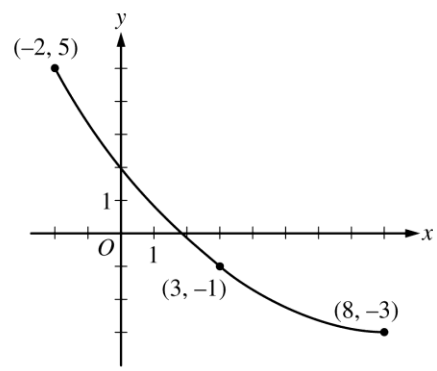
CALCULUS BC
WORKSHEET ON MEAN VALUE THEOREM

Work the following on **notebook** **paper**.

1. Let f be the function given by $f(x) = x^3 - 2x^2 + 5x - 16$. For what value of x in the closed interval $[0,5]$ does the instantaneous rate of change of f equal the average rate of change of f over that interval?

2. A portion of the graph of a differentiable function f is shown to the right. If the value $c = 3$ satisfies the conclusion of the Mean Value Theorem applied to f on the open interval $-2 < x < 8$, what is the slope of the line tangent to the graph of f at $x = 3$?



Graph of f

3. (Mult. Choice) The function g is continuous on the closed interval $[1,4]$ with $g(1) = 5$ and $g(4) = 8$. Of the following conditions, which would guarantee that there is a number c in the open interval $(1,4)$ where $g'(c) = 1$

(A) g is increasing on the closed interval $[1,4]$.
(B) g is differentiable on the open interval $(1,4)$.
(C) g has a maximum value on the closed interval $[1,4]$.
(D) The graph of g has at least one horizontal tangent in the open interval $(1,4)$.

4. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval $[0,3]$?

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x	–5	–4	–3	–2	–1	0
$g(x)$	10	5	2	3	1	0
$g'(x)$	–3	–1	4	1	–2	–3

5. Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x. Is there a number c in the closed interval $[-5,-3]$ such that $g'(c) = -4$? Justify your answer.

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t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

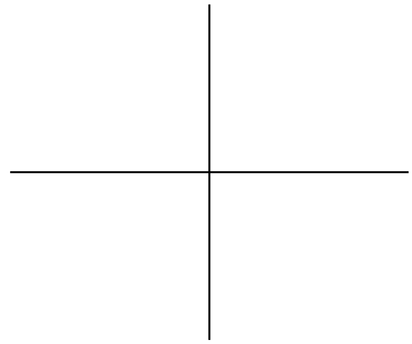
6. The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Explain why there must be at least one time t, for $2 < t < 10$, such that $H'(t) = 2$.

7. Given $f(x)=\frac{1}{3}x^3-x^2-3x+2$. $f'(x)=$ ______________________________

8. Use your calculator to graph $f(x)$ and $f'(x)$ in the following window:
$x:[-7.9,\ 7.9]$, $y:[-10,\ 10]$. Sketch below.

9. The relative maximum and minimum values of $f(x)$ occur at $x=$ ___________________

10. $f'(x)=$ 0 or $f'(x)$ is undefined at $x=$ ________________



11. $f(x)$ is increasing on what interval(s)? ________________

12. $f'(x)$ is positive on what interval(s)? ________________

13. $f(x)$ is decreasing on what interval(s)? ________________

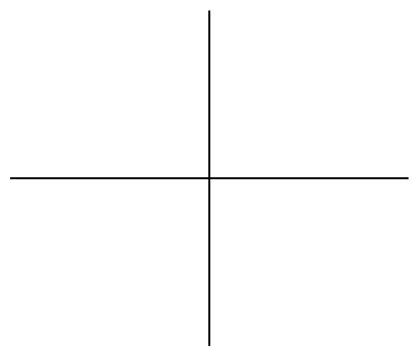
14. $f'(x)$ is negative on what interval(s)? ________________

15. Given $f(x)=(x-1)^{2/3}$. $f'(x)=$ ______________________________

16. Use your calculator to graph $f(x)$ and $f'(x)$ in the following window:
$x:[-1.975,\ 1.975]$, $y:[-2,\ 2]$. Sketch below.

17. The relative maximum and minimum values of $f(x)$ occur at $x=$ ___________________

18. $f'(x)=$ 0 or $f'(x)$ is undefined at $x=$ ________________



19. $f(x)$ is increasing on what interval(s)? ________________

20. $f'(x)$ is positive on what interval(s)? ________________

21. $f(x)$ is decreasing on what interval(s)? ________________

22. $f'(x)$ is negative on what interval(s)? ________________

Use your answers to problems 7 – 22 to complete the following statements:

23. The relative maximum and minimum values of f occurred when

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24. The function f is increasing when ____

25. The function f is decreasing when ____